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August 1989

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# Nuclear Fusion through Dimensional Confinement

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August 23, 1989

**Abstract**

A formal mechanism for enhancing nuclear fusion rates is proposed. The enhancement results whenever the reacting nuclei preferentially migrate in a restricted subspace of phase space—in particular, a fractal subspace. An extended Lawson criterion is derived, and the prospects for this mechanism in condensed matter are discussed.

**Keywords:** nuclear fusion, cold fusion, fractals, plasma confinement, quantum tunneling, transport properties of condensed matter

The results of recent experiments suggest the possibility of deuterium fusion (d-d reaction) occurring in certain metals (palladium in particular) at room temperature[1,2]. It is known that these metals are permeable to hydrogen, and that it accumulates at defects in the lattice[3]. The palladium is seasoned with deuterium by using it as a cathode in the electrolysis of heavy water. The question remains as to how the nuclei can overlap each other for a sufficient time to fuse.

The working hypothesis will be to assume that the fuel is restricted to move in a (generally) fractal subspace embedded in the material with a fractional dimension, $d^* < 3$. An asterisk on a variable will denote the generalization to the subspace of the corresponding 3-space quantity. A familiar example of a fractal subspace is the "snowflake" curve, which is embedded in two dimensions but has $d^* = \log 4/\log 3$. The result will be to increase the effective "density" of the deuterium, $n^*$, while decreasing the corresponding fusion "cross section," $\sigma^*$, by a lesser amount. This will result in a reduced mean free path for fusion, $\lambda = 1/(n^*\sigma^*)$.

For the time being, we will consider only the deuterium and the fractal space in which it is confined—*independent of any assumptions about the structure of the lattice in which it resides.* The following analysis is based on classical notions, but should be relevant to the quantum mechanical case as well. Justification and interpretation of the calculation may be made after the fact in ways similar to those made with renormalization and other formal mathematical methods.

Solids are manifestly three dimensional, i.e., extensive quantities scale like $r^d$, where $r$ is the sample diameter and $d = 3$. However, regions that are smaller than some crossover length, $L$, can have a fractional dimension, $d^*$, such that extensive quantities scale like $r^{d^*}$. In general, this multifractal behavior will have many scales and dimensions, but the essential features can be captured with two.

Let $N$ be the number of particles in a region with diameter $L$. Then the "densities" relevant to the fractal space satisfy

$$n^* = \frac{N}{V^*} = \frac{L^3 n}{L^{d^*}} = nL^{3-d^*}. \tag{1}$$

If we have an "interaction radius," $b$ (which is implicitly defined by equation (3) below), then the effective "cross section," $\sigma^*$, is essentially the "area" of a $d^* - 1$ dimensional disk and scales like

$$\sigma^* \propto b^{d^*-1}, \tag{2}$$

where the constant of proportionality depends on velocity and the details of the projection of the ordinary cross section, $\sigma$, onto the fractal space as seen by the moving particles.

Following Segrè[4], the power density due to the reaction of species 1 and 2 is

$$p_{12} = n_1 n_2 \langle \sigma v \rangle_{12} W_{12},$$

where $W_{12}$ is the energy produced per fusion. We want to generalize this formula to fractal spaces, and will specialize the discussion to a single type of reactant, viz., deuterium. Hence, we will drop the subscripts. The average interaction "volume" swept out per unit time by the relative velocity of two particles is $\langle \sigma^* v \rangle$, where the average is taken over a Maxwellian velocity distribution. The number of fusions per unit time for any deuteron is then $n^* \langle \sigma^* v \rangle$. Finally, the power "density" is given by

$$p^* = \frac{1}{2} n^{*2} \langle \sigma^* v \rangle W,$$

where the factor of 1/2 prevents double counting. In order to achieve the scientific break-even point, this power "density" must be maintained for a time, $\tau$, in order to supply a total energy "density" of $\frac{d^*}{2} n^* k T_i$ where $T_i$ is the ignition temperature, and $d^*$ is the number of degrees of freedom available to the plasma. Demanding that $p^* \tau$ exceed this gives

$$n^* \tau > \frac{d^* k T_i}{\langle \sigma^* v \rangle W}.$$

The usual Lawson criterion says that

$$n\tau > (n\tau)_{\min} = \frac{3kT_i}{\langle \sigma v \rangle W},$$

so the extended Lawson criterion is

$$\gamma n \tau > (n\tau)_{\min}.$$

Using equation (1) gives the boost factor

$$\gamma = \frac{3L^{3-d^*}}{d^*} \frac{\langle \sigma^* v \rangle}{\langle \sigma v \rangle} = \frac{3}{d^*} \left( \frac{L}{b} \right)^{3-d^*}, \tag{3}$$

which in turn sets the value of $b$ used in (2).

To see how the extra factor of $\gamma$ in the confinement parameter helps, let $\overline{\sigma} = 10^{-4}$ barns be a typical cross section and take $L = 1\,\mu$m. Then $b \cong \sqrt{\overline{\sigma}} = 10^{-14}$ cm, and

$$\gamma = \frac{3}{d^*} 10^{10(3-d^*)}.$$

This should be viewed against a standard value of $(n\tau)_{\min}$ of about $10^{14}\,\text{cm}^{-3}$s, assuming an ignition temperature of $T_i = 10\,$keV. One-dimensional confinement is the most favorable type to expect and gives a boost of twenty orders of magnitude! This would allow the ignition temperature to drop substantially. One might think that equation (3) actually favors a small cross section, but $b$ is fixed by the cross section at the given ignition temperature. The relevant parameters are the dimensionality, $d^*$, and the length scale, $L$. The interpretation of motion is rather strained for $d^* < 1$, but we should not dismiss the possibility. Fortunately, the enhancement diverges as $d^* \to 0$. This makes sense as the fuel becomes confined to discrete points.

In condensed matter, hydrogen densities would be on the order of $10^{24}\,\mathrm{cm}^{-3}$ to start with, but the thermal energies would be too small to overcome the Coulomb barrier. We would then need to operate in the pycnonuclear regime, where the fusion rate is very sensitive to density[5]. Jones et al. refer to this as piezonuclear fusion, and note that a reduction by a factor of two in atomic separation would enhance tunneling enough to explain their results[1]. Thus, for a saturated hydride, we would only need to increase the density by an order of magnitude. Suppose that the fractal space had a "thickness", $a$, where $a < L$, so that it was again three dimensional on the smallest scales. Then the new density would be

$$n' = \left(\frac{L}{a}\right)^{3-d^*} n.$$

Once again we see the dimensional deficit, $\varepsilon = 3 - d^*$, coming in. Here, $a$ might be on the order of a lattice spacing, $a = 10^{-8}\,\mathrm{cm}$. With the value of $L$ above, we would need a dimensionality of $d^* = 2.75$ to squeeze the deuterons together.

How might the motion of the deuterium be confined to a fractal space? On small scales, spatial restriction could be caused by dislocations which provide natural channels where the fuel might collect. Defects can be points, lines, or boundaries (and surfaces), so the most natural dimensions for these subspaces are zero, one, and two respectively. On large scales, it is conceivable that wave interactions with the lattice could cause momentum space restriction, even though each energy band forms a three-dimensional continuum. Deuterons, being bosons, can be squeezed into the same state, thus encouraging tunneling. Dimensional confinement, from any mechanism and to any extent, would enhance the fusion rate.

In actuality, the system would separate into a normal (three-dimensional) phase and a fractal phase. The Gibbs free energy for the fractal phase is

$$G = E - TS + P^*V^* = \mu N.$$

One would expect that the chemical potential, $\mu$, of the deuterium would be much higher in the restricted phase because of Coulomb repulsion and lowered entropy. But if the physics in the fractal space were sufficiently odd, the balance could be tipped to favor dimensional confinement. This would require a very strange equation of state indeed! It has been noted[1,2] that nonequilibrium processes (current flow in particular) are important, so the thermal character of a plasma might not be necessary. In any case, restricting the motion of the fuel is essential. All of this brings to mind superconductivity and superfluidity, where high order and unusual transport phenomena are found.

The thrust of this paper is not to explain the (inconclusive as of this writing) results of the current round of experiments, but to use this opportunity to suggest possible implications of dimensional confinement in condensed matter, particularly with respect to fusion. If this mechanism could be realized, it would provide an elegant method for harnessing fusion in the laboratory—in contrast to the chaotic, brute force approach taken in magnetic and inertial confinement reactors. Regardless of the outcome of these experiments, research into attaining fusion with phase space restriction and localization in condensed matter may yield promising results. Many new and surprising phenomena in condensed matter physics illustrate the potential of this program: superconductivity, quasi-crystals, organic and anisotropic conductors, laser driven particle acceleration, and the development of advanced electrochemical cells, just to name a few. Research and development in the semiconductor industry could play an important role because of the need to understand transport phenomena and to fabricate submicron structures and special compounds. Finally, the effects of dimensional confinement should be distinguished from any mechanism that may cause it. This could turn out to be a good way to think about the final theory of cold fusion. We eagerly await more experimental clues and theoretical developments to settle the question.

I would like to thank Eugene Gath for encouraging me to publish these ideas and John Baez and Michael Biafore for relaying news reports to me. I am indebted to the Information Mechanics Group in the Laboratory of Computer Science for the freedom to work on my own ideas. Support was provided by the National Science Foundation, grant no. IRI-8618002.

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